Cosmological Constant in the Thermodynamic Models of Gravity

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Abstract Within thermodynamic models of gravity, where the universe is considered as a finite ensemble of quantum particles, cosmological constant in the Einstein equations appears as a constant of integration. Then it can be bounded using Karolyhazy uncertainty relation applied for horizon distances, as the amount of information in principle accessible to an external observer.

Keywords: Cosmological constant, thermodynamic gravity, Karolyhazy uncertainty relation.

The cosmological constant problem (that its theoretical and observed values differ by many orders of magnitudes) is one of the biggest challenges in theoretical physics [1,2]. There have been several proposals to solve this discrepancy, like considering cosmological constant as a Lagrange multiplier, as a constant of integration, as a stochastic variable, anthropic interpretation, probabilistic interpretation and so on [2]. The effect of the cosmological constant can be also obtained through extended theories of gravity (see [3] and the references therein). In this article we will concentrate on thermodynamic models of gravity [4–13], where the cosmological constant arises in the Einstein’s equations as a constant of integration.

Black hole thermodynamics, the Unruh effect and some other evidences suggest that the gravitation has a fundamental connection to thermodynamics (see the reviews [11–13]). Within the thermodynamic model spacetime geometry emerges from the properties of the finite unified ensemble of quantum objects (particles - spacetime atoms). In this approach, the Einstein equations can be derived by combining general thermodynamic considerations with the equivalence principle and is written as a single scalar relation [11–14],

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = L_p^2 T_{\mu\nu}u^\mu u^\nu ,
\] (1)

where \(L_p\) is the reduced Planck length and \(u^\nu\) is the 4-velocity. This equation naturally can be interpreted as the balance of gravitational and matter heat densities, in the spirit of the first law of thermodynamics. Indeed, the right hand side of (1) can be regarded as the matter heat density, what is obvious, for example, for the case of ideal fluid using Gibbs-Duhem relation,

\[
T_{\mu\nu}u^\mu u^\nu \rightarrow \rho + P = TS ,
\] (2)

where \(T\) is the temperature and \(S\) is the entropy density of the fluid.

The scalar equation (1) involves additional vector field \(u^\nu\), but contains all information content of the ordinary tensorial Einstein equations, because it is demanded that it hold for all \(u^\nu\). In addition, if one assumes that \(u^\nu\) is an orthogonal to the observers horizon 4-velocity of light (null vector field) [11–13],

\[
g_{\mu\nu}u^\mu u^\nu = 0 ,
\] (3)

in the obtained from (1) tensorial Einstein equations,

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = L_p^2 (T_{\mu\nu} + g_{\mu\nu}\Lambda) ,
\] (4)

the cosmological constant, \(\Lambda\), arises as an integration constant, as in unimodular theories of gravity [15]. Unlike the thermodynamic approach, the unimodular models assume the constancy of the metric determinant in the variational problem without reasonable physical motivation.
If \( \Lambda \) in (4) is an integration constant, then it is not connected with the large constant vacuum energy terms in matter Lagrangians and needs to be fixed by an extra physical principle. For example, it can be identified with the amount of information accessible to an eternal observer at the horizon.

From quantum mechanics we know that information is a physical entity and must be inserted into the energy balance relations [16, 17]. At the same time, an information content of an isolated classical physical system usually is neglected. However, the entropy associated with observers could be important to describe properties of the entire universe [10]. So to find the numerical value of \( \Lambda \) we suggest to use the Károlyházy uncertainty relation [18], which bounds the ultimate precision of Minkowskian time measurement,

\[
\delta t \approx L_p^{2/3} t^{1/3}.
\]  

(5)

Due to this uncertainty relation, in the Minkowski space-time there exists a minimal cell, \( \delta t^3 \), whose energy cannot be smaller than

\[
E \sim \frac{1}{t}.
\]  

Then the energy density of the metric fluctuations of the Minkowski space-time is

\[
E \delta t^3 \sim \frac{1}{L_p^2 t^2}.
\]  

(7)

On the other hand, the vacuum energy density, \( \rho \), of an effective quantum field in a finite region with the length scale \( x \) cannot be arbitrary large, otherwise the region will collapse to a black hole. This implies that [19]

\[
\rho x^3 \leq \frac{x}{L_p^2},
\]  

(8)

which leads to

\[
\rho \sim \frac{1}{L_p^2 x^2}.
\]  

(9)

From the two estimations, (7) and (9), it follows that an observer at the horizon of a spatial region of the radius, \( x = t \), obtains some constant value of the internal vacuum energy density,

\[
\rho_{\text{min}} \equiv \Lambda = \text{const},
\]  

(10)

which can be understood as a measure of the quantum information deficit on the holographic screen around this region. For the case of the whole universe,

\[
t \sim x \sim \frac{1}{H},
\]  

(11)

where \( H \) is the present value of the Hubble constant, from the two estimations, (7) and (9), it follows that the minimal energy density in the universe (10), what can measure an external observer, is

\[
\Lambda \sim \frac{H^2}{L_p^2}.
\]  

(12)

This value coincides with the observed estimations for the dark energy [20, 21] and can be used to bound the integration constant \( \Lambda \) in the Einstein equations (4).

To conclude, in this small note we suggest identification of the cosmological constant (which within the thermodynamic model of gravity appears as a constant of integration) with the amount of information accessible to an external observer. Then its value can be obtained using the Károlyházy uncertainty relation applied to the universe, which gives the observed value of the dark energy.
References