# Optimal Selection of Dam Governance Scheme based on AHP and Multi-Objective Programming Model

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**Abstract.** For the dam management program, here are generally three options: the first to repair an existing dam, the second to rebuild a dam, the third type is to delete the original large dam and to replace the large dam with a series of small dams along the river.Based on Analytic Hierarchy Process (AHP), a model based on safety, cost, power generation, irrigation ability and ecological influence is established. The model is solved by using MATLAB and the consistency test is carried out. Finally, the weights are calculated and ranked according to AHP. The optimal solution was evaluated by Analytic Hierarchy Process (AHP).Then the case study of Kariba Dam is carried out. The results of the first and the third schemes are very close. Therefore, in order to further analyze the accuracy of the AHP results, we will focus on the comparison of schemes 1 and 3 with a detailed analysis of the third scheme. A multi-objective planning model with a minimum number of dams and a minimum construction cost is established. The model takes into account such factors as the amount of power generated, water capacity, and the number of dams. Finally, LINGO is used to solve the model. In data collection, some dams do not have accurate information, and regression equations are established by using known information to predict other information through EXCEL fitting, finally obtaining the optimal solution. Finally, the accuracy, applicability and flexibility of AHP results are verified.

**Keywords:** Analytic hierarchy process (AHP); multi-objective programming model; optimize management

## 1 Introduction

At present, many dams in the world have been in operation for many years. The erosion of the dam foundation has seriously threatened the stability of the dams. To effectively prevent such dams from being damped, the dams are urgently in need of maintenance. There are generally three alternatives for maintenance. The first is to repair existing dams, the second to rebuild dams to reshape reservoirs, and the third to remove existing dams and replace large dams with a series of small dams along the river. Although there are many technically large-scale dam rehabilitation programs, there is less research on the options of the programs. Due to the lack of effective economic, technological, and environmental assessment methods, the choice of programs in the past focused too much on technical feasibility. Based on the basic principles of the analytic hierarchy process, this paper establishes an evaluation model and proposes a judging system for large-scale dam rehabilitation programs. Taking Kariba Dam as an example, a comprehensive analysis of the multi-factors of large-scale dams, filters out the best plan.

## 2 Analytic Hierarchy Process Model

### 2.1 AHP General Principle

Analytic Hierarchy Process (AHP) is a multi-layer multi-objective decision making technique proposed by T. L. Satty. Mainly through the multi-factor decomposition of the system of complex problems to be evaluated, and based on the consistency judgment, integrate qualitative and quantitative issues, obtain the final quantitative evaluation index[1]. This can be used for the assessment and selection of dam management solutions. In the selection of these programs, there are many influencing factors, such as safety, cost, power generation capacity, irrigation capacity, ecological impact and so on. These factors are interrelated and mutually restrictive and constitute a complex system.

### 2.2 Building a Hierarchical Structure Model

According to system engineering and system level principle[2].After analyzing various influencing factors of dam governance, different factors are divided into different levels. Figure 1 shows the hierarchical model of the evaluation system for the Kariba dam treatment plan. It is divided into three layers: the first layer is the total target layer, the best solution for dam management evaluation. The second level is the sub-target level and is divided into five sub-goals: safety assessment, cost assessment, power generation capacity assessment, irrigation capacity assessment and ecological impact assessment. And according to the five evaluation indicators, write a judgment matrix as the sub-target layer. The third level is the program level, which is to repair existing dams for option one, the second program to build the dam, the third program to removes existing large dams. Figure 1 shows the analytic hierarchy diagram.

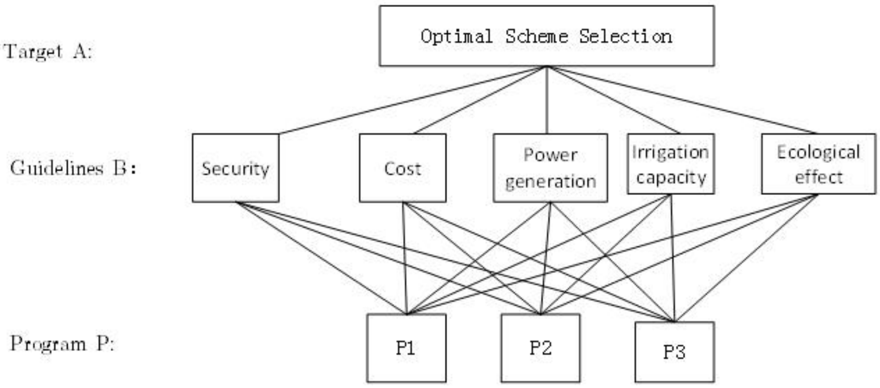


**Figure 1.** Analysis chart

### 2.3 Construction Judgment Matrix

Once the model structure is established, it is necessary to configure the weights for each factor[3]. This is a very important part, it is closely related to the specific engineering features and requirements. Even for the same model structure, engineering technology and environment, economic indicators require different weight be derived from the configuration. The configuration method strictly follows the AHP technology, specifically through the introduction of a suitable scale factor value, the formation of the judgment matrix. Generally according to a unified Saaty1 ~ 9 judging matrix standard table (Table1.), a judgment matrix A can be formed by comparing each pair of indicators, given the relative importance of the indicator, and given the corresponding index score. Table 2. shows the symbolic definition of AHP, and Table 3. shows the definition of evaluation index.

**Table 1.** Saaty1 ~ 9 judgment matrix standard degree

| scale | Definition |
|---|---|
| 1 | Two elements are equally important |
| 3 | The former is slightly more important than the latter |
| 5 | The former is obviously more important than the latter |
| 7 | The former is more important than the latter |
| 9 | The former is Extremely important than the latter |
| 2, 4, 6 , 8 | Median of the above adjudication |
| reciprocal | If the ratio of importance of i element to j is $b_{ij}$, the ratio of importance of j element to i $b_{ji} = 1 / b_{ij}$ |

 

**Table 2.** Symbols for Analytic Hierarchy Process

| Symbol | Definition |
|---|---|
| A | the judging matrix |
| $\lambda_{max}$ | the greatest eigenvalue of matrix A |
| CI | the indicator of consistency check |
| CR | the consistency ratio |
| RI | the random consistency index |
| CW | the weight vector for criteria level |
| AW | the weight vector for alternatives level |
| $Y_1$ | the evaluation grade for model I |

**Table 3.** Symbols for evaluation norms

| Symbol | Definition |
|---|---|
| $B_1$ | security |
| $B_2$ | cost |
| $B_3$ | Power generation |
| $B_4$ | Irrigation capacity |
| $B_5$ | ecological effect |

## 2.4 Hierarchical Ranking and Consistency Testing

- Determine the judging matrix

  We use the pairwise comparison method and one-nine method to construct judging matrix $A = \left(a_{ij}\right)$.

$$a_{ik} * a_{kj} = a_{ij} \tag{1}$$

  where air is set according to the one—nine method.

- Calculate the eigenvalues and eigenvectors

  The greatest eigenvalue of matrix A is $\lambda_{max}$, and the corresponding eigenvector is $u = \left(u_{1,} u_{2,} u_{3,} \cdots u_n\right)^T$. Then we normalize the u by the expression:

$$x_i = \frac{u_i}{\sum_{i=0}^{n} u_j} \tag{2}$$

- Do the consistency check

  The indicator of consistency check formula:

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{3}$$

  where n denotes the exponent number of matrix.

  The expression of consistency ratio:

$$CR = \frac{CI}{RI} \tag{4}$$

As we have confirmed the weighting coefficient of all the indicators in the evaluation system，now we quantify the importance of coaches.

$CW_i$ denotes the weight of $i^{th}$ criteria level factor, where $AW_j$ is the weight of $j^{th}$ secondary critical level factor.

The evaluation grade Y1 should be:

$$Y_1 = \sum_{i=1}^{5} CW_i * \sum_{j=1}^{5} AW_j \tag{5}$$

 

# 3 Project Examples

## 3.1 Project Overview

Kariba dam height of 128 meters, 617 meters long crest, the axis of the world's largest artificial reservoir - Kariba Lake, Zambezi on the Kariba dam is one of the larger dams in Africa[4]. Kariba dam has become the focus of regional energy security and economic development. The Kariba Dam completed its water storage from 1958 to 1963. Dam erosion has seriously threatened the stability of the dam. The Kariba reservoir dam has been in operation for more than 50 years. The spillway discharge has caused erosion to the basaltic dam foundation, forming deeper scouring pits and cutting into the dam foundation. A report by South Africa Institute of Risk Management 2015 warned that the dam is in urgent need of maintenance[5]. In order to effectively prevent the risk of dam break in Kariba, we should make the restoration of the Kariba dam foundation, the re-shaping of the reservoir, the renovation of the spillway and the improvement of the dam's operation, which are the main sources of the project-cost. The Zambezi River Authority (ZRA) now offers three options[6]: Repairing the existing Kariba Dam, Rebuilding the existing Kariba Dam, Removing the Kariba Dam and replacing it with a series of ten to fifteen.

In this paper, AHP will be used to evaluate the three schemes proposed by the Zambezi River Authority, and a multi-objective function programming model will be established for the third scheme. The model is selected and analyzed by LINGO to select the appropriate number of small dams.

## 3.2 Calculation Process Analysis

According to the unified Saaty1~9 level judgment matrix standard degree table (Table 1), by comparing the two indicators, the relative importance of the indicators is given, and the corresponding scores of the indicators are given. The following judgment matrix is derived.

- Judging matrix:

*Total target layer*

$$Judgment\ matrix\ A-B: \mathrm{A}=\begin{bmatrix} 1 & 3 & 3 & 4 & 1 \\ 1/3 & 1 & 1 & 1 & 1/3 \\ 1/3 & 1 & 1 & 1 & 1/3 \\ 1/4 & 1 & 1 & 1 & 1/4 \\ 1 & 3 & 3 & 4 & 1 \end{bmatrix}$$

*Sub − target layer*

$$Judgment\ matrix\ B-P: \quad \mathrm{B}_1=\begin{bmatrix} 1 & 1/4 & 1/3 \\ 4 & 1 & 2 \\ 3 & 1/2 & 1 \end{bmatrix} \quad \mathrm{B}_2=\begin{bmatrix} 1 & 3 & 1/3 \\ 1/3 & 1 & 1/6 \\ 3 & 6 & 1 \end{bmatrix}$$

$$\mathrm{B}_3=\begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 1/3 \\ 3 & 3 & 1 \end{bmatrix} \quad \mathrm{B}_4=\begin{bmatrix} 1 & 1/2 & 1/4 \\ 2 & 1 & 1/4 \\ 4 & 4 & 1 \end{bmatrix} \quad \mathrm{B}_5=\begin{bmatrix} 1 & 3 & 1/3 \\ 1/3 & 1 & 1/5 \\ 3 & 5 & 1 \end{bmatrix}$$

Using MATLAB programming (see Appendix 1 of the program) the results obtained are as follows.

- Weight vector of alternatives level: $CW = \left[0.3436\ 0.1081\ 0.0967\ 0.3436\right]$.

For this level, CI=0.0033, CR=0.0030, satisfying $\frac{CI}{RI} < 0.1$.

- Weight vector of alternatives level:

 

- ✓ Security: $AW_1 = \left[0.1220\ 0.5584\ 0.3196\right]$, For this level, CI=0.0091,CR=0.0158 , satisfying $\frac{CI}{RI} < 0.1$.
- ✓ Cost: $AW_2 = \left[0.2499\ 0.0953\ 0.6548\right]$ For this level, CI=0.0091, CR=0.0158 , satisfying $\frac{CI}{RI} < 0.1$.
- ✓ Power generation: $AW_3 = \left[0.1571\ 0.2493\ 0.5936\right]$, For this level, CI=0.0268, CR=0.0462, satisfying $\frac{CI}{RI} < 0.1$.
- ✓ Irrigation capacity: $AW_4 = \left[0.1311\ 0.2081\ 0.6608\right]$, For this level, CI=0.0268, CR=0.0462, satisfying $\frac{CI}{RI} < 0.1$.
- ✓ Ecological effect: $AW_5 = \left[0.7732\ 0.0877\ 0.1392\right]$, For this level, CI=0.0268, CR=0.0462, satisfying $\frac{CI}{RI} < 0.1$.

All of these three vectors satisfy $\frac{CI}{RI} < 0.1$

Finally, we can obtain the final ranking of the top three plan using AHP models.

**Table 4.** Total sorting table

| Criterion | | security | cost | generating capacity | Irrigation capacity | Ecological capacity | Total order weight | Rank |
|---|---|---|---|---|---|---|---|---|
| Criterion layer weight | | 0.3436 | 0.1081 | 0.1081 | 0.0967 | 0.3436 | | |
| Program layer single sort weight | ① | 0.1220 | 0.2499 | 0.1571 | 0.1311 | 0.7732 | 0.36427 | 1 |
| | ② | 0.5584 | 0.0953 | 0.2493 | 0.2081 | 0.0877 | 0.2790 | 3 |
| | ③ | 0.3196 | 0.6548 | 0.5936 | 0.6608 | 0.1392 | 0.3560 | 2 |

**Conclusion:**

From the above table, we can see that the total weight of the first strategy is 0.36427, second strategy is 0.2790, third strategy is 0.3560.

Due to the similar results obtained for options one and three, we will compare the potential costs and benefits of each strategy, and further analyze the accuracy of the analytic hierarchy process.

# 4 Program Three Analysis:

In the next section, we will proceed with a detailed modeling and analysis of scenario III. The third option is to remove the Kariba dam and replace it with a series of ten to twenty smaller dams along the Zambezi River. The reorganized new dam system would have the same overall water management capacity as existing Kariba dams while providing existing dams with the same or higher levels of protection and water management options as Lake Kariba.

In the detailed analysis of the third scheme, we establish a multi-objective planning function model using LINGO to solve. The sum of the generation capacities of all the small dams to be replaced shall be greater than or equal to the maximum capacity of the original Kariba dam and the sum of the water capacities of all the small dams to be replaced shall be greater than or equal to the maximum water capacity of the original Kariba dam. And the sum of all small dams should be greater than 10.

## 4.1 Multi-objective Linear Programming Model

**Symbol Description:**

In addition, we introduce a random variable $X_i$ to denote the i-th dam, $x_i$ is the i-th dam, i = 1, 2, 3, ..., 15, If the dam is chosen, 1 is taken and 0 if the dam is not selected. $W_i$ is the original capacity of the i-th dam, $\beta$ is the expansion coefficient, $M_i$ is the i-th dam power generation, $\alpha$ is the power generation increase coefficient, $C_i$ is the initial input cost of the i-th dam, $\gamma$ is the cost increase coefficient. The relevant literature and access to Wikipedia relevant information obtained $\alpha = 0.16$, $\beta = 0.13$, $\gamma = 0.16$.

The multiple goal linear programming model is presented below:

$$\min \sum_{i=1}^{15} x_i \tag{6}$$

$$\min \sum_{i=1}^{15} x_i * C_i * \Upsilon \tag{7}$$

Subject to:

$$\sum_{i=1}^{15} W_i * x_{i*} \beta \geq 180 \tag{8}$$

$$\sum_{i=1}^{15} M_i * x_i * \alpha \geq 1830 \tag{9}$$

$$\sum_{i=1}^{15} x_i \geq 10 \tag{10}$$

$$\alpha, \beta, \gamma > 0 \tag{11}$$

The objective function (6) is the least dam selected, and (7) is the least cost of the expansion. Constraints (8), All small dams after expansion have the same or higher level of overall water management capacity than existing Kariba dams. Constraints (9) The total power generation of all the small dams after expansion is greater than the current capacity of the existing Kariba dam. Constraints (10)The total number of all small dams is greater than 10.

## 4.2 Solve the Model

All the data are obtained by referring to the relevant websites and literature, but since some of the data did not find a definite content, we made a scatter plot fitting curve to calculate the relevant data. From Figure 2, we can see that the fitted curve between installed capacity and the total dam volume is an exponential function, from which we can calculate the total dam volume of six dams such as Victoria Falls South Bank hydropower station; From Figure 3 it can be seen that the fitted curve between installed capacity and the total dam value is a polynomial function, from which we can calculate the total value of 4 dams such as Dvur Gorge Hydropower Station based on its functional expression. See Appendix II for all data information.

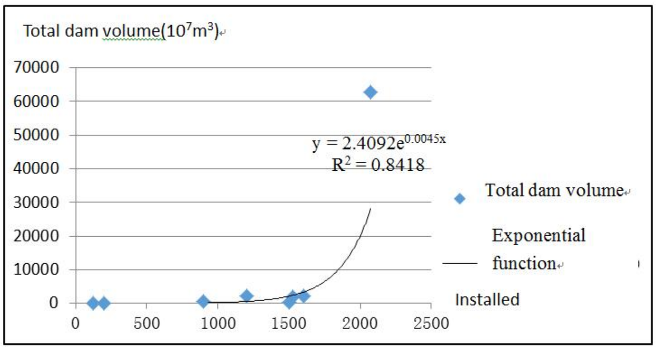


**Figure 2.** Scatter plot fitting curve of installed capacity and total dam volume

 

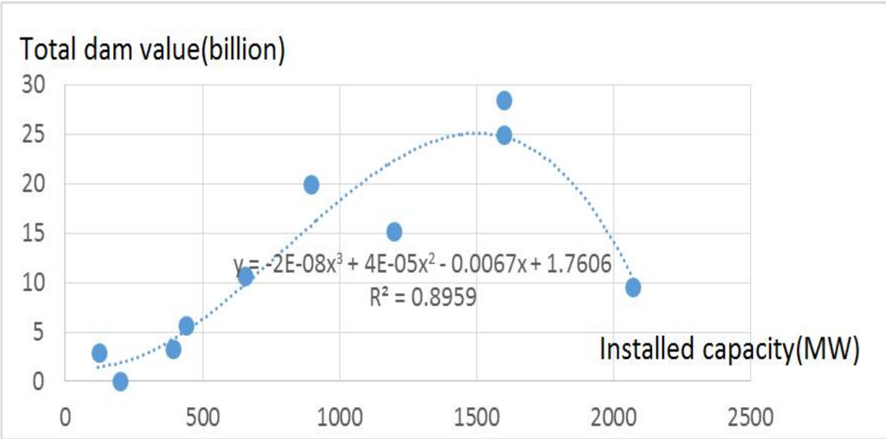


**Figure 3.** Scatter plot fitting curve of installed capacity and total dam value

### 4.3 Results

According to LINGO (see appendix III of the program), the software solved the model and found that if the Kariba Dam was deleted, it should be replaced by 10 small dams on the Zambezi River, The cost is 22.21923 billion dollars. The main result is shown in Figure 4, Figure 5.

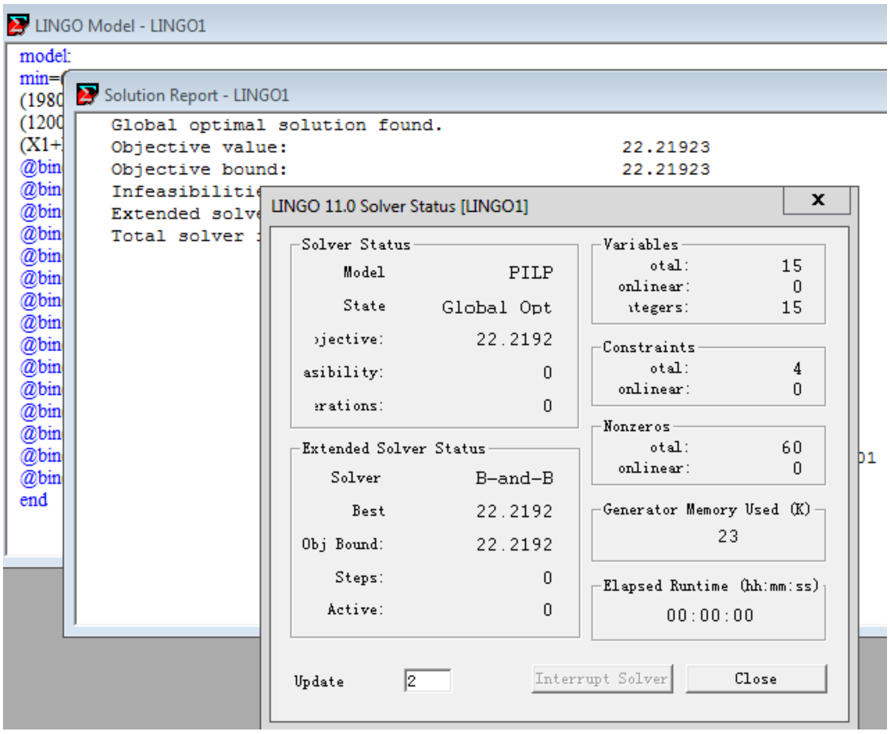


**Figure 4.** The calculation result of the cost of the dam

```
Solution Report - LINGO1
Global optimal solution found.
Objective value:                              10.00000
Objective bound:                              10.00000
Infeasibilities:                              0.000000
Extended solver steps:                               0
Total solver iterations:                             0


                       Variable           Value        Reduced Cost
                             X1        1.000000            1.000000
                             X2        0.000000            1.000000
                             X3        1.000000            1.000000
                             X4        1.000000            1.000000
                             X5        1.000000            1.000000
                             X6        1.000000            1.000000
                             X7        0.000000            1.000000
                             X8        1.000000            1.000000
                             X9        0.000000            1.000000
                            X10        1.000000            1.000000
                            X11        0.000000            1.000000
                            X12        1.000000            1.000000
                            X13        0.000000            1.000000
                            X14        1.000000            1.000000
                            X15        1.000000            1.000000

                            Row    Slack or Surplus      Dual Price
                              1        10.00000           -1.000000
                              2        9827.571            0.000000
                              3        360.4200            0.000000
```

**Figure 5**. The calculation result of the number of the dam

The 10 small dams are: Mupata Gorge Hydroelectric, Dvur Gorge Hydropower Station, Mphanada Nkuwa, Batoka Hydropower Station, Meipan David Card Station, Cabora Bassa Dam, Cartoon Nigeria reserrvior, Rome Hydropower station, Lupata Hydroelectric Power Plant, Zambiabia kafue Gorge Dam.The location on the map is shown in Figure6.


**Figure 6.** Location of small dams around the Kaliba dam (from Bigemap)

From the above analysis, we can get:

(1) On the cost side, if the Kariba dam was deleted and replaced with ten small dams along the Zambezi River in Scenario 3, the cost would be 22.21923 billion dollars, while the cost of repairing the Kariba dam[7] would not be more than 3 billion dollars, So the cost of a program lower than the cost of program three.

(2) In terms of the degree of ecological impact, the small dam along the Zambezi River will make the radiation radius of each dam larger and the surrounding ecological environment deteriorate. Therefore, the above factors come to a conclusion, that is, a more reasonable option.

 

## 5 Conclusion

Large-scale dam management programs need to be scientifically and reasonably evaluated. Choosing an appropriate dam management plan combining economic, technological and environmental aspects in engineering practice can not only ensure the quality of the project but also reduce the project cost. In this paper, the basic principles of AHP, established a hierarchical structure model for the selection of large-scale dam governance programs, and carried out hierarchical and quantitative processing of complex problems. The statistical analysis was incorporated into the entire process and a multi-objective programming model was established to accurately use lingo Solution to test the AHP result is the accuracy of the qualitative analysis and quantitative analysis of a better fusion method. The method has strong functionality, simple operation, easy popularization in engineering construction and strong practical value.

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## Appendix

### Appendix 1:

First program:
**Matlab**:

```
clc
a=[1,3,3,4,1
   1/3,1,1,1,1/3
   1/3,1,1,1,1/3
   1/4,1,1,1,1/4
   1,3,3,4,1];
[x,y]=eig(a);eigenvalue=diag(y);lamda=eigenvalue(1);
ci1=(lamda-5)/4;cr1=ci1/1.12
w1=x(:,1)/sum(x(:,1))
b1=[1,1/4,1/3;4,1,2;3,1/2,1];
[x,y]=eig(b1);eigenvalue=diag(y);lamda=eigenvalue(1);
ci21=(lamda-3)/2;cr21=ci21/0.58
w21=x(:,1)/sum(x(:,1))
b2=[1   3    1/3;1/3    1    1/6;3    6     1];
[x,y]=eig(b2);eigenvalue=diag(y);lamda=eigenvalue(1);
ci22=(lamda-3)/2;cr22=ci22/0.58
w22=x(:,1)/sum(x(:,1))
b3=[1     1/2   1/4;2   1     1/3;4    3    1];
[x,y]=eig(b3);eigenvalue=diag(y);lamda=eigenvalue(1);
ci23=(lamda-3)/2;cr23=ci23/0.58
```

```
w23=x(:,1)/sum(x(:,1))
b4=[1    1/2    1/4;2    1    1/3;4   3    1];
[x,y]=eig(b4);eigenvalue=diag(y);lamda=eigenvalue(1);
ci24=(lamda-3)/2;cr24=ci24/0.58
w24=x(:,1)/sum(x(:,1))
b5=[1    7    7;1/7    1    1/2;1/7   2    1];
[x,y]=eig(b5);eigenvalue=diag(y);lamda=eigenvalue(1);
ci25=(lamda-3)/2;cr25=ci25/0.58
w25=x(:,1)/sum(x(:,1))
w_sum=[w21,w22,w23,w24,w25]*w1
ci=[ci21,ci22,ci23,ci24,ci25];
cr=ci*w1/sum(0.58*w1)
```

Results:

```
cr1 =
0.0030
w1 =
    0.3436
    0.1081
    0.1081
    0.0967
    0.3436
cr21 =
  0.0158
w21 =
    0.1220
    0.5584
    0.3196
cr22 =
    0.0158
w22 =
    0.2499
    0.0953
    0.6548
cr23 =
    0.0158
w23 =
    0.1365
    0.2385
    0.6250
cr24 =
    0.0158
w24 =
    0.1365
    0.2385
    0.6250
cr25 =
    0.0462
w25 =
    0.7732
    0.0877
    0.1392
w_sum =
    0.3625
    0.2811
    0.3564
```

cr =
0.0262
Second procedures:
**First layer:** clc;

```
clear;
A=[1,3,3,4,1
   1/3,1,1,1,1/3
   1/3,1,1,1,1/3
   1/4,1,1,1,1/4
   1,3,3,4,1];
[m,n]=size(A);
RI=[0 0 0.58 0.90 1.12 1.24 1.32 1.41 1.45 1.49 1.51];
R=rank(A);
[V,D]=eig(A);
tz=max(D);
B=max(tz);
[row, col]=find(D==B);
C=V(:,col);
CI=(B-n)/(n-1);
CR=CI/RI(1,n);
if CR<0.10
    disp('CI=');disp(CI);
    disp('CR=');disp(CR);
    disp(': ');
    Q=zeros(n,1);
    for i=1:n
        Q(i,1)=C(i,1)/sum(C(:,1));
    end
    Q
else
    disp('');
end
```

**Results:**
CI=
0.0033
CR=
0.0030
By comparing the consistency of the matrix A, the vector weight vector Q:
Q =
0.3436
0.1081
0.1081
0.0967
0.3436

- $B_1, B_2, B_3, B_4, B_5$ Same as the first layer.

**Appendix 2:**

The data in Table 4.2.1are compiled from the relevant websites and literatures.

**Table 4.2.1:** Data on 15 small dams along the Zambezi River

| Name of Dam | installed capacity (MW) | Total volume of dam ($10^7m^3$) | value (Billion) |
|---|---|---|---|
| MUPATA GORGE HYDROELECTRIC | 1200 | 1980 | 15.16 |

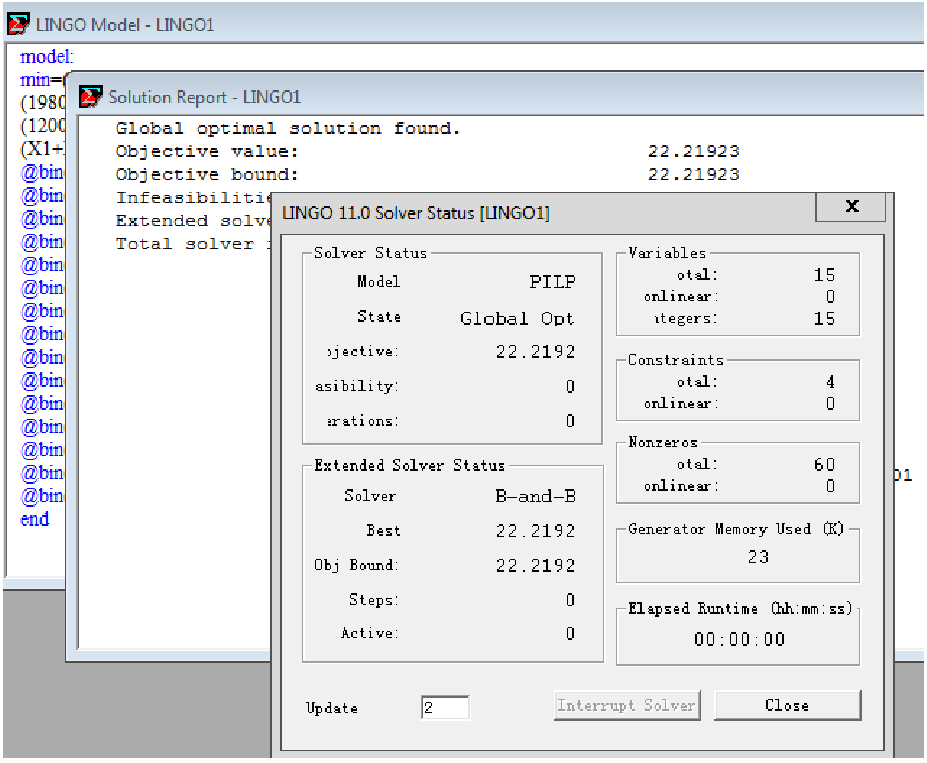
| | | | |
|---|---|---|---|
| Itezhitezhi Hydropower Station | 120 | 8.5 | 2.75 |
| Dvur Gorge Hydropower Station | 1526 | 1680 | 13.613 |
| Mphanada Nkuwa | 1600 | 250 | 28.497 |
| Batoka Hydropower Station | 1600 | 2000 | 24.89 |
| Meipan David Card Hydropower Station | 1780 | 7253.9 | 27.626 |
| kamuzu barrage | 200 | 1.3 | 0.0762 |
| Cahora Bassa Dam | 2075 | 630008 | 9.5 |
| Victoria Falls South Bank hydropower station | 390 | 13.933 | 3.24 |
| Cartoon Nigeria reserrvior | 390 | 13.933 | 4.05 |
| Hydro Power Station in Tedzani Falls | 40 | 2.884 | 1.56 |
| Rome Hydropower station | 444 | 17.77 | 5.725 |
| NkulaFalls Hydropower Station | 60 | 3.16 | 1.498 |
| Lupata Hydroelectric Power Plant | 654 | 45.71 | 10.72 |
| Zambiabia kafue Gorge Dam | 900 | 740 | 20 |

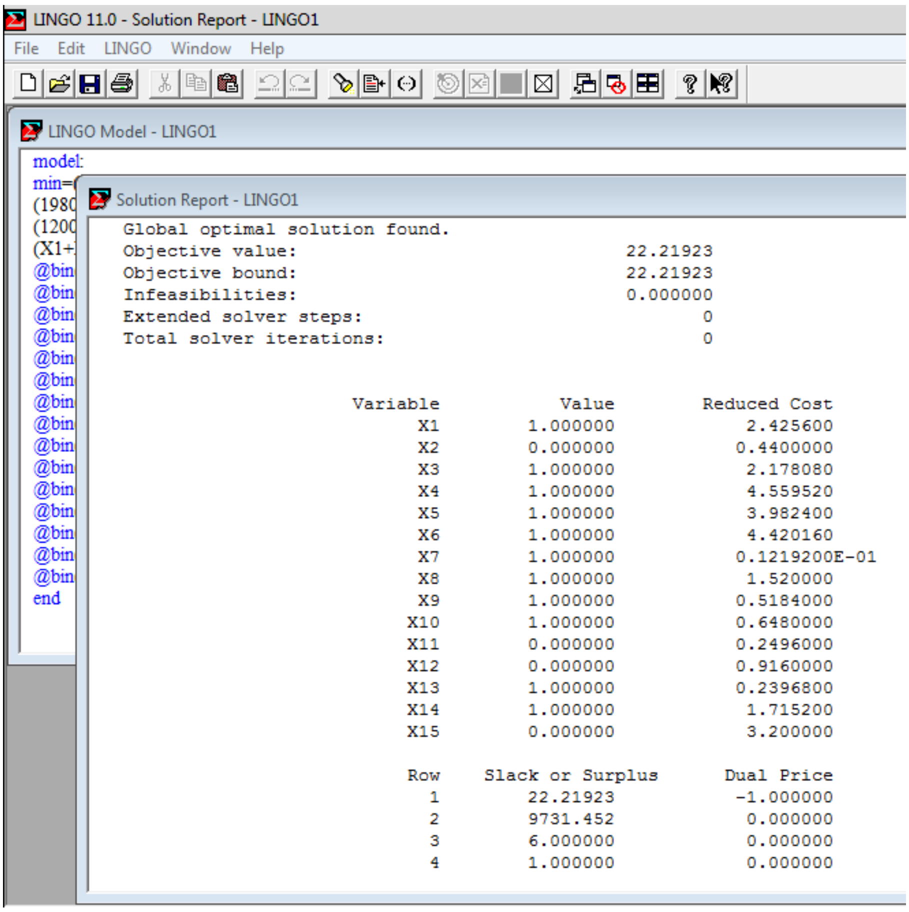
**Appendix 3:**

Minimum cost:

```
model:
min=(15.16*X1+2.75*X2+13.613*X3+28.497*X4+24.89*X5+27.626*X6+0.0762*X7+9.5*X8+3.24*X9
+4.05*X10+1.56*X11+5.725*X12+1.498*X13+10.72*X14+20*X15)*0.16;
(1980*X1+8.5*X2+1680*X3+250*X4+2000*X5+7253.9*X6+1.3*X7+63000*X8+13.933*X9+13.933*X1
0+2.884*X11+17.77*X12+3.16*X13+45.71*X14+740*X15)*0.13>=180;
(1200*X1+120*X2+1526*X3+1600*X4+1600*X5+1780*X6+200*X7+2075*X8+390*X9+390*X10+40*
X11+444*X12+60*X13+654*X14+900*X15)*0.16>=1830;
(X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+X13+X14+X15)>=10;
@bin(X1);
@bin(X2);
@bin(X3);
@bin(X4);
@bin(X5);
@bin(X6);
@bin(X7);
@bin(X8);
@bin(X9);
@bin(X10);
@bin(X11);
@bin(X12);
@bin(X13);
@bin(X14);
@bin(X15);
end
```


LINGO Model - LINGO1
model:
end
Solution Report - LINGO1
Global optimal solution found.
Objective value: 22.21923
Objective bound: 22.21923
LINGO 11.0 Solver Status [LINGO1]
Solver Status
Model PILP
State Global Opt
Extended Solver Status
Solver B-and-B
Best 22.2192
Obj Bound: 22.2192
Steps: 0
Active: 0
Variables
15
0
15
Constraints
4
0
Nonzeros
60
0
Generator Memory Used (K)
23
Elapsed Runtime (hh:mm:ss)
00:00:00
Update
2
Interrupt Solver
Close


LINGO 11.0 - Solution Report - LINGO1
File Edit LINGO Window Help
LINGO Model - LINGO1
model:
end
Solution Report - LINGO1
Global optimal solution found.
Objective value: 22.21923
Objective bound: 22.21923
Infeasibilities: 0.000000
Extended solver steps: 0
Total solver iterations: 0
Variable Value Reduced Cost
X1 1.000000 2.425600
X2 0.000000 0.4400000
X3 1.000000 2.178080
X4 1.000000 4.559520
X5 1.000000 3.982400
X6 1.000000 4.420160
X7 1.000000 0.1219200E-01
X8 1.000000 1.520000
X9 1.000000 0.5184000
X10 1.000000 0.6480000
X11 0.000000 0.2496000
X12 0.000000 0.9160000
X13 1.000000 0.2396800
X14 1.000000 1.715200
X15 0.000000 3.200000
Row Slack or Surplus Dual Price
1 22.21923 -1.000000
2 9731.452 0.000000
3 6.000000 0.000000
4 1.000000 0.000000

**Minimum number of dams:**

```
model:
min=X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+X13+X14+X15;
(1980*X1+8.5*X2+1680*X3+250*X4+2000*X5+7253.9*X6+1.3*X7+63000*X8+13.933*X9+13.933*X1
0+2.884*X11+17.77*X12+3.16*X13+45.71*X14+740*X15)*0.13>=180;
(1200*X1+120*X2+1526*X3+1600*X4+1600*X5+1780*X6+200*X7+2075*X8+390*X9+390*X10+40*
X11+444*X12+60*X13+654*X14+900*X15)*0.18>=1830;
(X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+X13+X14+X15)>=10;
(15.16*X1+2.75*X2+13.613*X3+28.497*X4+24.89*X5+27.626*X6+0.0762*X7+9.5*X8+3.24*X9+4.05*
X10+1.56*X11+5.725*X12+1.498*X13+10.72*X14+20*X15)*0.16 > 22.21923;
@bin(X1);
@bin(X2);
@bin(X3);
@bin(X4);
@bin(X5);
@bin(X6);
@bin(X7);
@bin(X8);
@bin(X9);
@bin(X10);
@bin(X11);
@bin(X12);
@bin(X13);
@bin(X14);
@bin(X15);
end
```