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Quantum Entanglement and Magnetic Moment of Deuteron

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Abstract. It is argued that the exact representation of the magnetic moment of deuteron can be achieved when the effect of quantum entanglement among the nearest neighbour constituents of nucleons is taken into account.

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One of the fundamental problems in nuclear physics is the exact representation of the magnetic moment of deuteron. The experimental value of deuteron magnetic moment (in unit of nucleon magneton) is given as

$$\mu_d = 0.857456$$

whereas the simple sum of the magnetic moments of the constituent nucleons yields

$$\mu_p + \mu_n = 0.880$$

where μ_p, μ_n are the magnetic moments of proton and neutron respectively. As evident from the above data, the deviation of the magnetic moment of deuteron from the simple sum of μ_p and μ_n is about 0.0225. To reduce the deviation 4% D-state probability in the deuteron wave function was considered with a phenomenological approach [1]. However, such a consideration is subject to important corrections arising out of some physical facts as follows. Firstly the contribution to the magnetic moment by mesons which are exchanged between the nucleolus is to be considered. Secondly, the relativistic corrections for the nucleonic motion should be taken into account. Thirdly, possible modifications in the proton magnetic moment due to the presence of the neutron and vice versa, should also be considered. Taking together all these factors, the current experimental data favour 6-7 % D-state probability in the deuteron wave function to make up the deviation. In a recent study [2], it has been pointed out that all possible corrections via relativistic quantum mechanics on a possible mixture of S, D and P states can reduce the above deviation by about 1%. A similar situation exists in a way to represent the magnetic moments of tritium, helium and other such few-nucleon systems.

In the present note we shall address the problem from the viewpoint of quantum entanglement among the constituents of nucleons. In the paper ref. [3] has been pointed out that the quantization of a formation can be achieved when we introduce an internal variable which appears as a direction vector (vortex line) depicting spin degrees of freedom. A vortex line is topologically equivalent to a magnetic flux line. In this scenario, a fermion is represented as a skyrmion which can be viewed as a scalar particle attached with a magnetic flux line (vortex line) which gives rise to the spin degrees of freedom. Indeed the localization region of a relativistic massless as well as massive particle is found to be S^3 [4,5]. Since S^3 is equivalent to the group $SU(2)$ and $S^2=SU(2)/U(1)$, the sphere S^3 can be constructed from the 3D compact space S^2 by Holf fibration. The Abelian field $U(1)$ corresponds to monopole bundle which gives rise to the spin degrees of freedom. The entanglement of two vortices can be viewed as that of two spins. The entanglement of two identical particles studied at two delocalized regions can be considered to be caused by the interaction of two magnetic flux lines associated with the corresponding vortices and then get separated. Thus the measure of entanglement given by concurrence can be considered to be related to the monopole strength. Indeed in some recent papers [6,7] it has been shown that concurrence essentially corresponds to the monopole strength associated with the vortex line corresponding to the spin.

In this context we may add that the Dirac quantization condition suggests that the monopole charge μ takes the values 0, ±1/2, ±1, ±3/2…… However in an entangled spin system the monopole charge undergoes a renormalization group (RG) flow [8-11].

The rules for the RG flow of the monopole charge known as the μ-theorem are as follows. When the monopole charge depends on a certain parameter λ we have

1) μ is stationary at fixed points λ^* of the RG flow
2) At the fixed points $\mu(\lambda^*)$ is equal to the monopole charge given by quantized values (μ=0, ±1/2, ±1……)
3) μ decreases along the RG flow i.e. $L(\partial\mu/\partial L)\leq 0$ where L is a scale parameter.

In view of this analysis we now note that the nonquantized angular momentum associated with the RG flow of the monopole charge when it is mapped onto an antiferromagnetic spin chain. Indeed Bouchiat and Me'zard [12] have pointed out that the writhe partition function Fourier transform corresponds to the quantum mechanical problem of a charged particle in the field of a monopole with nonquantized charge.

A quantum particle attains a geometric phase (Berry Phase) when it traverses a closed path enclosing a magnetic flux line [13]. The phase is given by $2\pi\mu$, μ being the monopole strength. When the monopole is located at the origin of a unit sphere, the phase is given by $\mu\Omega$(C) where Ω(C) is the solid angle subtended by the closed contour at the origin and is given by

$$\Omega(C) = \int_c (1-\cos\theta)d\phi = 2\pi(1-\cos\theta) \tag{1}$$

where θ is polar angle of the vortex line measured from the quantization axis (z-axis). For a spin ½ particle μ=1/2 and so the Berry phase is given by

$$\phi_B = \pi(1-\cos\theta) \tag{2}$$

In some earlier papers [6, 7] it has been pointed out that in a spin system the measure of entanglement of two nearest neighbour spins given by concurrence C can be expressed as

$$C = \left|\phi_B / 2\pi\right| \tag{3}$$

ϕ_B being the Berry Phase acquired by the state when rotated in a closed path. Indeed equating ϕ_B given by π (1−cosθ) with $2\pi\mu$, we find

$$\mu = (1/2)(1-\cos\theta) \tag{4}$$

This corresponds to the monopole strength associated with the corresponding vortex line and represents the measure of entanglement given by concurrence of two nearest neighbour spins in a spin system.

In an entangled spin system the ground state is characterised by the accumulated, Berry phase

$$\phi_B = \sum_N \pi(1-\cos\theta) \tag{5}$$

and the concurrence stored in the system is given as

$$C = \sum_N C(N) = \sum_N 1/2(1-\cos\theta) \tag{6}$$

where N is the number of nearest neighbour pairs.

Now to study the deviation of the deuteron magnetic moment in terms of concurrence associated with the entanglement of the nearest neighbour vortices of the constituents of nucleons, we take resort to the skyrmionic picture of a nucleon. Skyrme [14] first introduced the idea that a nucleon may be described by a nonlinear σ - model with the basic pionic degrees of freedom when the baryon number is found to be topological origin. Later on, Adkins, Nappi and Witten [15] have studied the static properties of nucleons in terms of this skyrme model and their results have shown almost 30% discrepancy with the experimental values. In an earlier paper [16] we have studied the geometrical properties of skyrmions and have pointed out that the internal $\delta U(3)$ symmetry of hadrons can be derived from reflection group when we consider a hadron as a composite state of skyrmions. These skyrmions are constructed from pionic degrees of freedom when a pion is attached with a magnetic flux line (vortex line). The attachment of the magnetic flux line changes the angular momentum of the pion and endows orbital spin to it. In fact in presence of a magnetic monopole μ the orbital angular momentum of a charged particle is changed through the relation $\vec{J} = \vec{L} - \mu\hat{r}$ where $\vec{L}$ is the orbital momentum and $\hat{r}$ is a unit vector. Thus we note that for $\vec{L} = 0$, μ=1/2 and so the pion attains the orbital spin $\vec{J} = 1/2$. In this context it has been suggested that we can consider a nucleon as a composite state of two such skyrmions with a spin carrier attached to it and the static properties derived from this picture are found to be in good agreement with the experimental values [16]. In this scenario, a nucleon can be considered to be composed of two pionic degrees of freedom attached with a spin carrier such that each constituent pion is represented by a skyrmion when these move in an anisotropic space (a fictitious magnetic field) with the orbital angular momentum l=1/2 and the two constituents have opposite l_z values. This effectively comprises the orbital spin of the constituent skyrmions. It is noted that in the strong interaction region where we have changes independence we can neglect the charge of the pions concerned and consider them as identical particles. We can now consider the entanglement of nearest neighbour orbital spins of the skyrmions. In a deuteron there is a chain of 4 orbital spins in an antiferromagnetic array where each orbital spin corresponds to the l_z value of the massive constituent represented by a skyrmion. Thus the system represents a Heisenberg spin chain. It has been found that in the thermodynamic limit, for a linear antiferromagnetic chain, the concurrence due to the entanglement between the nearest neighbour spins is given by C=0.386 [17]. In a finite chain the value will be changed slightly from this. In a system of 4 orbital spins arranged in an antiferromagnetic array the total concurrence stored in the chain due to the bipartite entanglement of nearest neighbour orbital spins will be

$$C_d = \sum C(N) = 3/2(1-\cos\theta) \tag{7}$$

which follows from eqn (6).

This is related to the deviation of monopole strength $|\Delta\mu|$ from its value when the constituent nucleons are free and we have

$$|\Delta\mu| = (3/2)\cos\theta \tag{8}$$

Now to study the magnetic moment of deutorn in unit of nuclear Bohr magneton we have to take into account the fact that in a skyrmionic structure there is pionic degree of freedom and so the corresponding mass of the pion is to be changed incorporating the mass of the nucleon. Thus we write for deviation of the magnetic moment from the sum of μ_p and μ_n

$$\Delta\mu_d = 3/2(\cos\theta)(m_\pi / 2m_N) \tag{9}$$

Now with the value of the concurrence for the entanglement of a pair of nearest neighbour spins in a linear antiferromagnetic chain, viz. C=1/2(1−cosθ)=0.386, we find from the relation (9)

$$\Delta\mu_d = 0.025 \tag{10}$$

The value obtained in this way in found to be excellent agreement with the experimental value 0.0225. The slight discrepancy may be due to the correction for the finite number of sites from the value of C=0.386 which is valid in the thermodynamic limit.

From the present analysis, it is found that quantum entanglement is responsible for the deviation of the exact value of the magnetic moment of deuteron from the simple sum of proton and neutron magnetic moments. A similar analysis can be carried out in other few −nucleon systems.

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