On The Class of Almost $\beta$-$\gamma$-Continuous Functions

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Abstract The main purpose of the present paper is to introduce a new class of functions called almost $\beta$-$\gamma$-continuous functions which is contained in the class of almost $\beta$-continuous functions and contains the class of $\beta$-$\gamma$-continuous functions.

Keywords: $\beta$-$\gamma$-open, almost $\beta$-$\gamma$-continuous.

1 Introduction

Kasahara [10] defined an operation $\alpha$ on a topological space to introduce $\alpha$-closed graphs. Following the same technique, Ogata [16] defined an operation $\gamma$ on a topological space and introduced $\gamma$-open sets. Hariwan [7] introduced a type of continuity called $\beta$-$\gamma$-continuous function. Nasef and Noiri [13] introduced the notion of almost $\beta$-continuity.

In this paper, we introduce a new class of functions called almost $\beta$-$\gamma$-continuous functions which is contained in the class of almost $\beta$-continuous functions and contains the class of $\beta$-$\gamma$-continuous functions. We obtain basic properties of almost $\beta$-$\gamma$-continuous functions.

2 Preliminaries

Throughout this paper, $(X, \tau)$ and $(Y, \sigma)$ stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset $A$ of $X$, the closure of $A$ and the interior of $A$ will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let $(X, \tau)$ be a space and $A$ a subset of $X$. An operation $\gamma$ [10] on a topology $\tau$ is a mapping from $\tau$ into power set $P(X)$ of $X$ such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of $\gamma$ at $V$. A subset $A$ of $X$ with an operation $\gamma$ on $\tau$ is called $\gamma$-open [16] if for each $x \in A$, there exists an open set $U$ such that $x \in U$ and $\gamma(U) \subseteq A$. Then, $\gamma_1$ denotes the set of all $\gamma$-open set in $X$. Clearly $\gamma_1 \subseteq \tau$. Complements of $\gamma$-open sets are called $\gamma$-closed. The $\gamma_1$-interior [18] of $A$ is denoted by $\gamma_1-Int(A)$ and defined to be the union of all $\gamma$-open sets of $X$ contained in $A$. A subset $A$ of a space $X$ is said to be $\beta$-$\gamma$-open [8] if $A \subseteq Cl(\gamma_1-Int(Cl(A)))$. A subset $A$ of $X$ is called $\beta$-$\gamma$-closed [7] if and only if its complement is $\beta$-$\gamma$-open.

Definition 2.1. A subset $A$ of a space $X$ is said to be

1. $\alpha$-open [14] if $A \subseteq Int(Cl(Int(A)))$.
3. preopen [12] if $A \subseteq Int(Cl(A))$.
4. $\beta$-open [1] if $A \subseteq Cl(Int(Cl(A)))$.

Definition 2.2. The intersection of all preclosed (resp., semi-closed, $\alpha$-closed) sets of $X$ containing $A$ is called the preclosure [6] (resp., semi-closure [4], $\alpha$-closure [17]) of $A$.

Definition 2.3. [19] The $\delta$-interior of a subset $A$ of $X$ is the union of all regular open sets of $X$ contained in $A$. The subset $A$ is called $\delta$-open if $A = Int_\delta(A)$, i.e., a set is $\delta$-open if it is the union of regular open sets. The complement of a $\delta$-open set is called $\delta$-closed. Alternatively, a set $A \subseteq X$ is called $\delta$-closed if $A = Cl_\delta(A)$, where $Cl_\delta(A) = \{x \in X : Int(Cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

Proposition 2.4. [2] A subset $A$ of a space $X$ is $\beta$-open if and only if $Cl(A)$ is regular closed.
Theorem 2.5. [1] Let $A$ be any subset of a space $X$. Then $A \in \beta O(X)$ if and only if $Cl(A) = Cl(Inter(Cl(A)))$.

Theorem 2.6. Let $A$ be a subset of a topological space $(X, \tau)$. Then:
1. If $A \in SO(X)$, then $pCl(A) = Cl(A)$ [5].
2. If $A \in \beta O(X)$, then $\alpha Cl(A) = Cl(A)$ [3].
3. If $A \in \beta O(X)$, then $Cl_\delta(A) = Cl(A)$ [20].

Lemma 2.7. [9] Let $A$ be a subset of a space $(X, \tau)$. Then $A \in O(X, \tau)$ if and only if $sCl(A) = Int(Cl(A))$.

Definition 2.8. Let $A$ be any subset of a topological space $(X, \tau)$ and $\gamma$ be an operation on $\tau$. Then:
1. The union of all $\beta-\gamma$-open sets contained in $A$ is called the $\beta-\gamma$-interior of $A$ and is denoted by $\beta-\gamma Int(A)$.
2. The intersection of all $\beta-\gamma$-closed sets containing $A$ is called the $\beta-\gamma$-closure of $A$ and is denoted by $\beta-\gamma Cl(A)$.

Definition 2.9. [7] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\beta-\gamma$-continuous if for every open set $V$ of $Y$, $f^{-1}(V)$ is $\beta-\gamma$-open in $X$.

Definition 2.10. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $\beta-\gamma$-continuous if for each $x \in X$ and each open set $V$ of $Y$ containing $f(x)$, there exists a $\beta-\gamma$-open set $U$ containing $x$ such that $f(U) \subseteq V$.

Definition 2.11. [13] A function $f : (X, \tau) \to (Y, \sigma)$ is called almost $\beta$-continuous at a point $x \in X$ if for every open set $V$ in $Y$ containing $f(x)$, there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq Int(Cl(V))$. If $f$ is almost $\beta$-continuous at every point of $X$, then it is called almost $\beta$-continuous.

Definition 2.12. [15] A space $X$ is said to be semi-regular if for any open set $U$ of $X$ and each point $x \in U$, there exists a regular open set $V$ of $X$ such that $x \in V \subseteq U$.

3 Almost $\beta-\gamma$-Continuous

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called almost $\beta-\gamma$-continuous at a point $x \in X$ if for each $x \in X$ and each open set $V$ of $Y$ containing $f(x)$, there exists a $\beta-\gamma$-open set $U$ of $X$ containing $x$ such that $f(U) \subseteq Int(Cl(V))$. If $f$ is almost $\beta-\gamma$-continuous at every point of $X$, then it is called almost $\beta-\gamma$-continuous.

Example 3.2. Consider $X = \{1, 2, 3\}$ with the discrete topology $\tau$ on $X$. Define an operation $\gamma$ on $\tau$ by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{1, 3\} \\ X & \text{otherwise.} \end{cases}$$

And define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} 1 & \text{if } x = 1 \\ 2 & \text{if } x = 2 \\ 3 & \text{if } x = 3 \end{cases}$$

Then, $f$ is not $\beta-\gamma$-continuous.

Remark 3.3. It easily follows that $\beta-\gamma$-continuity implies almost $\beta-\gamma$-continuity and almost $\beta-\gamma$-continuity implies almost $\beta$-continuity. However, the converses are not true as the following example shows.

Example 3.4. Consider $X = \{a, b, c\}$ with the topology $\tau = \sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define an operation $\gamma$ on $\tau$ by $\gamma(A) = A$ for all $A \in \tau$. Define a function $f : (X, \tau) \to (X, \sigma)$ as follows:

$$f(x) = \begin{cases} c & \text{if } x = a \\ b & \text{if } x = b \\ a & \text{if } x = c \end{cases}$$
Then $f$ is almost $\beta$-continuous but not $\beta$-continuous, because $\{a\}$ is an open set in $(X, \sigma)$ containing $f(c) = a$, but there exists no $\beta$-open set $U$ in $(X, \tau)$ containing $c$ such that $f(U) \subseteq \{a\}$.

And we define an operation $\gamma$ on $\tau$ by $\gamma(A) = X$ for all $A \in \tau$. Then $f$ is almost $\beta$-continuous but is not almost $\beta$-continuous.

**Theorem 3.5.** For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:

1. $f$ is almost $\beta$-continuous.
2. For each $x \in X$ and each open set $V$ of $X$ containing $f(x)$, there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq sCl(V)$.
3. For each $x \in X$ and each regular open set $V$ of $Y$ containing $f(x)$, there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq \gamma(V)$.
4. For each $x \in X$ and each $\delta$-open set $V$ of $Y$ containing $f(x)$, there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq V$.

**Proof.** (1) $\Rightarrow$ (2). Let $x \in X$ and let $V$ be any open set of $Y$ containing $f(x)$. By (1), there exists a $\beta$-open set $U$ of $X$ containing $x$ such that $f(U) \subseteq \gamma(V)$ in $V$. Since $V$ is open and hence $V$ is preopen set. By Lemma 2.7, $\gamma(V) = sCl(V)$. Therefore, $f(U) \subseteq sCl(V)$.

(2) $\Rightarrow$ (3). Let $x \in X$ and let $V$ be any regular open set of $Y$ containing $f(x)$. Then $V$ is an open set of $Y$ containing $f(x)$. By (2), there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq sCl(V)$.

Since $V$ is regular open and hence is preopen set. By Lemma 2.7, $sCl(V) = \gamma(V)$. Therefore, $f(U) \subseteq \gamma(V)$.

(3) $\Rightarrow$ (4). Let $x \in X$ and let $V$ be any $\delta$-open set of $Y$ containing $f(x)$. Then for each $f(x) \in V$, there exists an open set $\gamma$ containing $f(x)$ such that $\gamma \subseteq \gamma(V)$. Since $\gamma(V)$ is regular open set of $Y$ containing $f(x)$. By (3), there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq \gamma(V)$.

This completes the proof.

(4) $\Rightarrow$ (1). Let $x \in X$ and let $V$ be any open set of $Y$ containing $f(x)$. Then $\gamma(V)$ is $\delta$-open set of $Y$ containing $f(x)$. By (4), there exists a $\beta$-open set $U$ in $X$ containing $x$ such that $f(U) \subseteq \gamma(V)$.

Therefore, $f$ is almost $\beta$-continuous. □

**Theorem 3.6.** For a function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:

1. $f$ is almost $\beta$-continuous.
2. $f^{-1}(\gamma(V))$ is $\beta$-open set in $X$, for each open set $V$ in $Y$.
3. $f^{-1}(\gamma(U))$ is $\beta$-open set in $X$, for each closed set $U$ in $Y$.
4. $f^{-1}(U)$ is $\beta$-open set in $X$, for each regular closed set $U$ of $Y$.
5. $f^{-1}(U)$ is $\beta$-open set in $X$, for each regular open set $V$ of $Y$.

**Proof.** (1) $\Rightarrow$ (2). Let $V$ be any open set in $Y$. We have to show that $f^{-1}(\gamma(V))$ is $\beta$-open set in $X$. Let $x \in f^{-1}(\gamma(V))$. Then $f(x) \in \gamma(V)$ and $\gamma(V)$ is a regular open set in $Y$. Since $f$ is almost $\beta$-continuous. Then by Theorem 3.5, there exists a $\beta$-open set $U$ of $X$ containing $x$ such that $f(U) \subseteq \gamma(V)$. This implies that $x \in U \subseteq f^{-1}(\gamma(V))$. Therefore, $f^{-1}(\gamma(V))$ is $\beta$-open set in $X$.

(2) $\Rightarrow$ (3). Let $F$ be any closed set of $Y$. Then $f^{-1}(\gamma(V)) = f^{-1}(\gamma(U))$. Since $F$ is a closed set of $X$ and hence $f^{-1}(\gamma(U))$ is $\beta$-open set in $X$.

(3) $\Rightarrow$ (4). Let $F$ be any regular closed set of $Y$. Then $f^{-1}(\gamma(U)) = f^{-1}(U)$ is $\beta$-closed set in $X$. Since $F$ is regular closed set. Then $f^{-1}(\gamma(U)) = f^{-1}(F)$. Therefore, $f^{-1}(F)$ is $\beta$-closed set in $X$.

(4) $\Rightarrow$ (5). Let $V$ be any regular open set of $Y$. Then $f^{-1}(V) = f^{-1}(U)$ is $\beta$-closed set in $X$ and hence $f^{-1}(V)$ is $\beta$-open set in $X$.

(5) $\Rightarrow$ (1). Let $x \in X$ and let $V$ be any regular open set of $Y$ containing $f(x)$. Then $x \in f^{-1}(V)$. By (5), we have $f^{-1}(V)$ is $\beta$-open set in $X$. Therefore, we obtain $f(f^{-1}(V)) \subseteq V$. Hence by Theorem 3.5, $f$ is almost $\beta$-continuous. □

**Theorem 3.7.** For a bijection function $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:
Theorem 3.9. Proof. (1) \Rightarrow (2). Let A be a subset of X. Since $\text{Cl}_f(A)$ is $\delta$-closed in Y, it is denoted by $\cap \{F_\alpha : F_\alpha \in \text{RC}(Y), \alpha \in \Delta\}$, where $\Delta$ is an index set. Then, we have $A \subseteq f^{-1}(\text{Cl}_f(A)) = f^{-1}(\cap \{F_\alpha : \alpha \in \Delta\})$. By (1) and Theorem 3.6, $f^{-1}(\text{Cl}_f(A))$ is $\beta\gamma$-closed set of X. Hence $\beta\gamma\text{Cl}(A) \subseteq f^{-1}(\text{Cl}_f(A))$. Therefore, we obtain $f(\beta\gamma\text{Cl}(A)) \subseteq \text{Cl}_f(A)$. 

(2) \Rightarrow (3). Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (2), we have $f(\beta\gamma\text{Cl}(f^{-1}(B))) \subseteq \text{Cl}_f(B)$. Hence $\beta\gamma\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(\text{Cl}_f(B))$. 

(3) \Rightarrow (4). Let F be any $\delta$-closed set of Y. By (3), we have $\beta\gamma\text{Cl}(f^{-1}(F)) \subseteq f^{-1}(\text{Cl}_f(F)) = f^{-1}(F)$ and hence $f^{-1}(F)$ is $\beta\gamma$-closed set in X.

(4) \Rightarrow (5). Let V be any $\delta$-closed set of Y. Then $Y \setminus V$ is $\delta$-closed set of Y and by (4), we have $f^{-1}(Y \setminus V) = \beta\gamma\text{Cl}(V)$ is $\beta\gamma$-closed set in X. Hence $f^{-1}(V)$ is $\beta\gamma$-open set in X. 

(5) \Rightarrow (6). For each subset B of Y. We have $\text{Int}_\alpha(B) \subseteq B$. Then $f^{-1}(\text{Int}_\alpha(B)) \subseteq f^{-1}(B)$. By (5), $f^{-1}(\text{Int}_\alpha(B)) \subseteq \beta\gamma$-open set in X. Then $f^{-1}(\text{Int}_\alpha(B)) \subseteq \beta\gamma\text{Int}(f^{-1}(B))$.

(6) \Rightarrow (7). Let A be any subset of X. Then $f(A)$ is a subset of Y. By (6), we obtain that $f^{-1}(\text{Int}_\alpha(f(A))) \subseteq \beta\gamma\text{Int}(f^{-1}(f(A)))$. Hence $f^{-1}(\text{Int}_\alpha(f(A))) \subseteq \beta\gamma\text{Int}(A)$, which implies that $\text{Int}_\alpha(f(A)) \subseteq f(\beta\gamma\text{Int}(A))$. 

(7) \Rightarrow (1). Let x be any regular open set of Y containing f(x). Then x is $\beta\gamma$-open in X. 

Theorem 3.8. For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:

1. $f$ is almost $\beta\gamma$-continuous.
2. $\beta\gamma\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$, for each $\beta\gamma$-open set V of Y.
3. $f^{-1}(\text{Int}(F)) \subseteq \beta\gamma\text{Int}(f^{-1}(F))$, for each $\beta\gamma$-closed set F of Y.
4. $f^{-1}(\text{Int}(F)) \subseteq f^{-1}(\text{Cl}(F))$, for each semi-closed set F of Y.
5. $\beta\gamma\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$, for each semi-closed set V of Y.

Proof. (1) \Rightarrow (2). Let V be any $\beta\gamma$-open set of Y. It follows from Proposition 2.4, that $\text{Cl}(V)$ is regular closed set in Y. Since $f$ is almost $\beta\gamma$-continuous. Then by Theorem 3.6, $f^{-1}(\text{Cl}(V))$ is $\beta\gamma$-closed set in X. Therefore, we obtain $\beta\gamma\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$.

(2) \Rightarrow (3). Let F be any $\beta\gamma$-closed set of Y. Then $Y \setminus F$ is $\beta\gamma$-open set of Y and by (2), we have $\beta\gamma\text{Cl}(f^{-1}(Y \setminus F)) \subseteq f^{-1}(\text{Cl}(Y \setminus F)) \Rightarrow \beta\gamma\text{Cl}(X \setminus f^{-1}(F)) \subseteq f^{-1}(Y \setminus \text{Int}(F)) \Rightarrow X \setminus \beta\gamma\text{Int}(f^{-1}(F)) \subseteq X \setminus f^{-1}(\text{Int}(F))$. Therefore, $f^{-1}(\text{Int}(F)) \subseteq \beta\gamma\text{Int}(f^{-1}(F))$.

(3) \Rightarrow (4). This is obvious since every semi-closed set is $\beta\gamma$-closed set.

(4) \Rightarrow (5). Let V be any semi-closed set of Y. Then $Y \setminus V$ is semi-closed set and by (4), we have $f^{-1}(\text{Int}(Y \setminus V)) \subseteq \beta\gamma\text{Int}(f^{-1}(Y \setminus V)) \Rightarrow f^{-1}(Y \setminus \text{Cl}(V)) \subseteq \beta\gamma\text{Int}(X \setminus f^{-1}(V)) \Rightarrow X \setminus f^{-1}(\text{Cl}(V)) \subseteq X \setminus \beta\gamma\text{Cl}(f^{-1}(V))$. Therefore, $\beta\gamma\text{Cl}(f^{-1}(V)) \subseteq f^{-1}(\text{Cl}(V))$.

(5) \Rightarrow (1). Let F be any regular closed set of Y. Then F is semi-closed set of Y. By (5), we have $\beta\gamma\text{Cl}(f^{-1}(F)) \subseteq f^{-1}(\text{Cl}(F)) = f^{-1}(F)$. This shows that $f^{-1}(F)$ is $\beta\gamma$-closed set in X. By Theorem 3.6, $f$ is almost $\beta\gamma$-continuous.
Theorem 3.11. A function \( f : X \to Y \) is almost \( \beta, \gamma \)-continuous if and only if \( f^{-1}(V) \subseteq \beta, \gamma \text{Int}(f^{-1}(\text{Cl}(V))) \) for each preopen set \( V \) of \( Y \).

Proof. **Necessity.** Let \( V \) be any preopen set of \( Y \). Then \( V \subseteq \text{Int}(\text{Cl}(V)) \) and \( \text{Int}(\text{Cl}(V)) \) is regular open set in \( Y \). Since \( f \) is almost \( \beta, \gamma \)-continuous, by Theorem 3.6, \( f^{-1}(\text{Int}(\text{Cl}(V))) \) is \( \beta, \gamma \)-open set in \( X \) and hence we obtain that \( f^{-1}(V) \subseteq f^{-1}(\text{Int}(\text{Cl}(V))) = \beta, \gamma \text{Int}(f^{-1}(\text{Int}(\text{Cl}(V)))) \).

**Sufficiency.** Let \( V \) be any regular open set of \( Y \). Then \( V \) is preopen set of \( Y \). By hypothesis, we have \( f^{-1}(V) \subseteq \beta, \gamma \text{Int}(f^{-1}(\text{Int}(\text{Cl}(V)))) = \beta, \gamma \text{Int}(f^{-1}(V)) \). Therefore, \( f^{-1}(V) \) is \( \beta, \gamma \)-open set in \( X \) and hence by Theorem 3.6, \( f \) is almost \( \beta, \gamma \)-continuous. \( \square \)

Corollary 3.12. A function \( f : X \to Y \) is almost \( \beta, \gamma \)-continuous if and only if \( f^{-1}(V) \subseteq \beta, \gamma \text{Int}(f^{-1}(\text{sCl}(V))) \) for each preopen set \( V \) of \( Y \).

Corollary 3.13. A function \( f : X \to Y \) is almost \( \beta, \gamma \)-continuous if and only if \( \beta, \gamma \text{Cl}(f^{-1}(\text{Cl}(\text{Int}(F)))) \subseteq f^{-1}(F) \) for each preclosed set \( F \) of \( Y \).

Corollary 3.14. A function \( f : X \to Y \) is almost \( \beta, \gamma \)-continuous if and only if \( \beta, \gamma \text{Cl}(f^{-1}(\text{sInt}(F))) \subseteq f^{-1}(F) \) for each preclosed set \( F \) of \( Y \).

Theorem 3.15. For a function \( f : X \to Y \), the following statements are equivalent:

1. \( f \) is almost \( \beta, \gamma \)-continuous.
2. For each neighborhood \( V \) of \( f(x) \), \( x \in \beta, \gamma \text{Int}(f^{-1}(\text{sCl}(V))) \).
3. For each neighborhood \( V \) of \( f(x) \), \( x \in \beta, \gamma \text{Int}(f^{-1}(\text{Int}(\text{Cl}(V)))) \).

Proof. Follows from Theorem 3.11 and Corollary 3.12. \( \square \)

Theorem 3.16. Let \( f : X \to Y \) is an almost \( \beta, \gamma \)-continuous function and let \( V \) be any open subset of \( Y \). If \( x \in \beta, \gamma \text{Cl}(f^{-1}(V)) \) \( \setminus f^{-1}(V) \), then \( f(x) \in \beta, \gamma \text{Cl}(V) \).

Proof. Let \( x \in X \) such that \( x \in \beta, \gamma \text{Cl}(f^{-1}(V)) \setminus f^{-1}(V) \) and suppose \( f(x) \notin \beta, \gamma \text{Cl}(V) \). Then there exists a \( \beta, \gamma \)-open set \( H \) containing \( f(x) \) such that \( H \cap V = \emptyset \). Then \( \text{Cl}(H) \cap V = \emptyset \) which implies \( \text{Int}(\text{Cl}(H)) \cap V = \emptyset \) and \( \text{Int}(\text{Cl}(H)) \) is regular open set. Since \( f \) is almost \( \beta, \gamma \)-continuous, by Theorem 3.5, there exists a \( \beta, \gamma \)-open set \( U \) in \( X \) containing \( x \) such that \( f(U) \subseteq \text{Int}(\text{Cl}(H)) \). Therefore, \( f(U) \cap V = \emptyset \). However, since \( x \in \beta, \gamma \text{Cl}(f^{-1}(V)), U \cap f^{-1}(V) \neq \emptyset \) for every \( \beta, \gamma \)-open set \( U \) in \( X \) containing \( x \), so that \( f(U) \cap V \neq \emptyset \). We have a contradiction. It follows that \( f(x) \in \beta, \gamma \text{Cl}(V) \). \( \square \)

Theorem 3.17. If \( f : X \to Y \) is almost \( \beta, \gamma \)-continuous and \( g : Y \to Z \) is continuous and open. Then the composition function \( \text{gof} : X \to Z \) is almost \( \beta, \gamma \)-continuous.
Proof. Let \( x \in X \) and \( W \) be an open set of \( Z \) containing \( g(f(x)) \). Since \( g \) is continuous, \( g^{-1}(W) \) is an open set of \( Y \) containing \( f(x) \). Since \( f \) is almost \( \beta\gamma \)-continuous, there exists a \( \beta\gamma \)-open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq \text{Int}(\text{Cl}(g^{-1}(W))) \). Also, since \( g \) is continuous, then we obtain \( (gof)(U) \subseteq g(\text{Int}(g^{-1}(\text{Cl}(W)))) \). Since \( g \) is open, we obtain \( (gof)(U) \subseteq \text{Int}(\text{Cl}(W)) \). Therefore, \( gof \) is almost \( \beta\gamma \)-continuous. \( \square \)

Theorem 3.18. If \( f : X \to Y \) is an almost \( \beta\gamma \)-continuous function and \( Y \) is semi-regular, then \( f \) is \( \beta\gamma \)-continuous.

Proof. Let \( x \in X \) and let \( V \) be any open set of \( Y \) containing \( f(x) \). By the semi-regularity of \( Y \), there exists a regular open set \( G \) of \( Y \) such that \( f(x) \in G \subseteq V \). Since \( f \) is almost \( \beta\gamma \)-continuous. By Theorem 3.5, there exists a \( \beta\gamma \)-open set \( U \) of \( X \) containing \( x \) such that \( f(U) \subseteq G \subseteq V \). Therefore, \( f \) is \( \beta\gamma \)-continuous. \( \square \)

References

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